Basic Quantum Mechanics
PH2201

Notes based on lectures for PH2201 (Basic Quantum Mechanics)
at IISER Kolkata by Professor Golam Mortuza Hossain, Spring 2024

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Contents

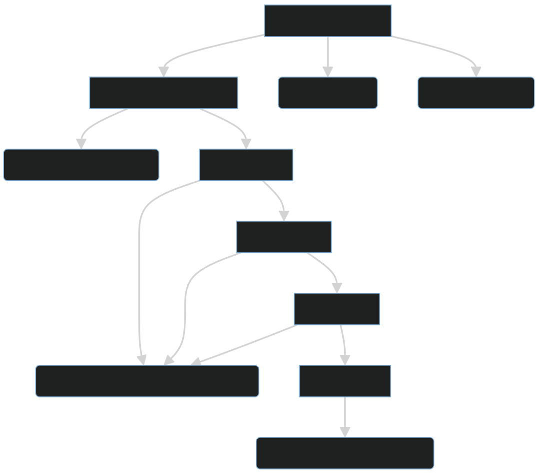
1. Lecture 1

- Evaluation was discussed
- What is classical physics?
    - It follows NLM, is macroscopic in nature, etc.

Nothing else was covered in this lecture apart from this.

2. Lecture 2

Here is a summary of the broad history of classical physics -



In summary, the most important feature was the law of motion, encapsulated by the equation -

$$m\frac{d^2\vec{r}}{dt^2} = \vec{F} \quad [1]$$

(assuming that the mass is not time variant). We may also note the fact that that expression for the electromagnetic force -

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \quad [2]$$

which , in the *static limit*, is

$$\vec{F} = q\vec{E} \quad [3]$$

and the expression for the gravitational force -

$$\begin{gathered} \vec{F} = m\vec{g} \\ \vec{g} = -\frac{GM\,\hat{}\,(r)}{r^2} \end{gathered} \quad [4]$$

Look quite **similar**.

There were thus two questions that came -
- Why is the speed of light an universal constant?
  - This was solved by **Albert Einstein** with his **Special Theory of Relativity**.
- Why is there no velocity dependent term here for gravitation - something like a magnetic field compoent?
  - This was again resolved by **Albert Einstein** in his **General Theory of Relativity** - in which he establishes that Gravity is essentially just an effect of being in the wrong reference frame.

This is essentially all of classical physics.

3. Lecture 3

4. Lecture 4

5. Lecture 5

6. Lecture 6

7. Lecture 7 (15 Jan 2024)

We begin with the question of determinism.

- How do we measure the position (say, x_0) of a particle?

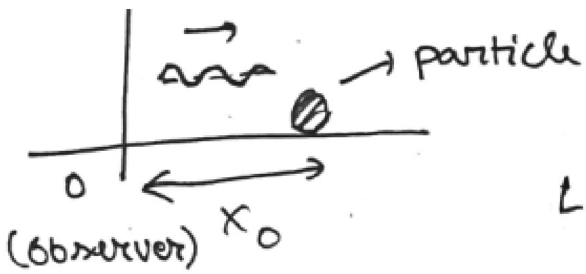


Figure 1: Demonstration of how the position of a particle is measured

What we essentially do, is send a photon to the particle and count the time that it takes for the photon to come back. We then use this information, coupled with the speed of light, to calculate the position of the particle.

But there is a problem with this method.

Light itself has a finite extent, i.e. a wavelength of light stretches over a finite length. When the light reflects from the particle, there must be a node there at the particle. But the problem arises with the fact that due to the finite width of the wavelength of light - we do not know which node of the full wavelength is there at the particle. This is, in general, a basic fault of all sensors - they can not detect the light until a full wavelength of the light is through.

As a consequence of this, we get the uncertainty in measurement -

$$\Delta x \propto \frac{\lambda}{2} \qquad [5]$$

This is an inherent error in the measurement of the position of the particle. This suggests that the *smaller* is the wavelength of the light that we use for the measurement, the *lower* is the error.

But it turns out that even this gives rise to another problem.

- How do we measure the velocity (say, v_0) of a particle?

We essentially send two light pulses pulses to the particle - each separated by a fixed time interval (say, t_0), and then measure the time that each takes to come back. From these two measurements of position (say, x_0 and ${x_0}'$) and the fixed time interval between the measurements, we can determine the velocity of the particle.

$$v_0 = \frac{x_0 - {x_0}'}{\frac{t_0}{2}} \qquad [6]$$

But the problem arises form the fact that the photon itself has some momentum.

$$p = \frac{h}{\lambda} \qquad [7]$$

Since the photon gets reflected from the particle and comes back, the particle gains some momentum. There is thus some inherent error in the measurement of the momentum.

$$\begin{gathered} \frac{h}{\lambda} - p_1 = -\frac{h}{\lambda} + p_2 \\ \Rightarrow \Delta p \propto \frac{2h}{\lambda} \end{gathered} \qquad [8]$$

Now, this suggests that we should use a *larger* wavelength to *reduce* the error in momentum measurement (and hence the measurement of the velocity).

Combining the effects of Equation 5 and Equation 8, we get -

$$\Delta x \Delta p \propto h \qquad [9]$$

Which suggests that this particular quantity is *independent of the measurement itself.* This is often referred to as the **first primitive form of the Heisenberg Uncertainty Principle**.

Thus, we may conclude that **classical mechanics depends on a condition that cannot be provided**.

$\Rightarrow x$ and p are *not* good variables to describe the quanta.

7.1. Vector Calculus

Vector — Definition 7.1.1

An element of a set, known as a linear vector space is called a **vector**.

Incomplete lecture

8. Lecture 8 (17 Jan 2024)

We begin by recalling the eigenvalue equation.

Eigenvalue equation — Definition 8.1

An equation of the form

$$\boldsymbol{O}\psi = \lambda\psi \quad [10]$$

is called an eigenvalue equation, given that $\boldsymbol{O}$ is an operator and ψ is a vector that it acts upon, called the **eigenvector** in this case. λ is called an **eigenvalue** the operator.

Maxwell's wave equation — Definition 8.2

$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial x^2}$$

Where we have,

$$\psi \to \vec{E}, \vec{B}$$

and this has a solution of the general form,

$$\psi = \psi(t, \vec{x}) = \psi_0 e^{i\left(\vec{k}\cdot\vec{x} - \omega t\right)} \quad [11]$$

Which is an EM wave that is travelling with the angular frequency ω and in the direction of the wave-vector k.

Further, this wave-function ψ also describes a quanta of energy, $E = h\nu = \hbar\omega$.

Schroedinger had to formulate a new way to read off the energy of a wave by combining the eigenvalue method and Planck's hypothesis. He was of the opinion that we are probably not reading physics in the proper way.

Consider the following action, where we use Equation 11

$$i\hbar\frac{\partial\psi}{\partial t} = (i\hbar)(-i\omega)\psi$$
$$\Rightarrow i\hbar\frac{\partial\psi}{\partial t} = \hbar\omega\psi$$
$$\Rightarrow i\hbar\frac{\partial\psi}{\partial t} = E\psi = \widehat{H}\psi$$

This is an eigenvalue equation where the eigenvalue is the energy of the light quanta. Therefore, the operator, $\hat{O} = i\hbar\frac{\partial}{\partial t}$ represents the energy operator - which is usually called the **Hamiltonian operator** or $\widehat{H}$.

$$i\hbar\frac{\partial\psi}{\partial t} = \widehat{H} \qquad [12]$$

Similarly, we may be able to deduce that the momentum operator for the wave-function is given by,

$$\hat{p} = -i\hbar\vec{\nabla} \qquad [13]$$

As we know that $p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = hk$.

We use this method to invert the question, and use this to derive physics.

8.1. Bohr's Model

The energy of the electron is given by,

$$E = \frac{p^2}{2m} + V$$

Where p is the momentum of the electron, m is the mass of the electron, and V is the potential at which the electron is.

Now, the Hamiltonian operator corresponding to the electron is,

$$\widehat{H} = \frac{\hat{p}^2}{2m} + \hat{V} = -\frac{\hbar^2}{2m}\nabla^2 + V \qquad [14]$$

Like the quanta of light, the electron should also be described by some *wave-function*, such that the following holds -

$$i\hbar\frac{\partial\psi}{\partial t} = \widehat{H}\psi$$

8.2. Schroedinger's Equation

Schroedinger's Equation — Definition 8.2.1

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = \widehat{H}\psi \qquad [15]$$

Let us assume that this was given to us in a dream. PDEs like this are not easy to solve in general. We will employ the method of **separation of variables** to solve this.

8.3. Separation of variables

We assume the intial *ansatz* of

$$\Psi(\vec{x}, t) = T(t)\psi(\vec{x})$$

We apply this to the Schroedinger equation in (1+1) dimensions.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

Plugging in the *ansatz*,

$$\Rightarrow \frac{1}{T(t)}i\hbar\frac{dT(t)}{dt} = \frac{1}{\psi(x)}\left[-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)\right] = E = \text{constant}$$

As,

- The potential is not taken to be a function of time.
- The LHS is a function of time, and the RHS is a function of space only. If this relation is to hold for all space and time, it has to equate to constant that is neither a function of time, nor space.

Thus, we arrive at the **Time dependent part of the Schroedinger equation**,

$$i\hbar \frac{dT}{dt} = ET \qquad [16]$$

and the **Time independent part of the Schroedinger equation**,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x) \qquad [17]$$

References